University of BRISTOL

Bristol Composites Institute

An Entropy-Based Perspective on Fracture Toughness

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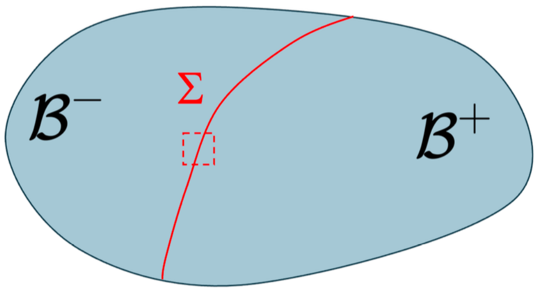
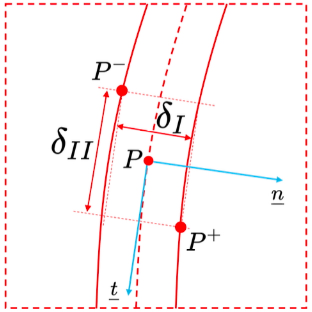
Entropy and Fracture Toughness





Principles of Thermodynamics (I+II):

$$\dot{s} = \frac{\dot{h}}{T} - \frac{\dot{q}}{T_B} + \dot{\gamma}, \quad \dot{\gamma} \geq 0.$$

Dissipation:

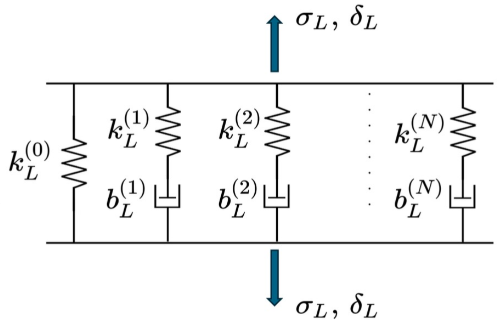
$$T\dot{\gamma} = \left[\sigma - \frac{\partial f}{\partial \delta}\right]\dot{\delta} - \left(s + \frac{\partial f}{\partial T}\right)\dot{T} + Y\dot{D} + \dot{q}\left(\frac{T}{T_B} - 1\right), \quad Y := -\frac{\partial f}{\partial D}.$$

Helmholtz free energy for **Generalised Maxwell model:**

$$f = \frac{1}{2}(1-D)\left[k_0(T)\delta^2 + \sum_{n=1}^{N} k_n \int_{0^-}^{t}\int_{0^-}^{t} e^{-\frac{2\hat{t}(t)-\hat{t}(\xi_1)-\hat{t}(\xi_2)}{\tau_n}} d\delta(\xi_1) d\delta(\xi_2)\right] - \int_{T_B}^{T}\int_{0}^{\theta_2} \frac{c(\theta_1)}{\theta_1} d\theta_1 d\theta_2 - (T - T_B)s_1(D)$$

Coleman-Noll -> $\dot{\gamma} = \frac{1}{T} Y\dot{D} + \dot{q}\left(\frac{1}{T_B} - \frac{1}{T}\right)$

Work to failure -> $w_f = T_B\gamma_f + \int_{0^-}^{t_f} \dot{h}(t)\left[\frac{T(t)}{T_B} - 1\right] dt + \int_{T_B}^{T_f} c(\theta_1)\left(1 - \frac{T_B}{\theta_1}\right) d\theta_1.$



Rate- and Temperature-Dependent Viscoelastic CZM

A <u>physically-based</u> model that includes

- Strain rate effects at the interfaces due to QS/dynamic loading
- Temperature and moisture effects

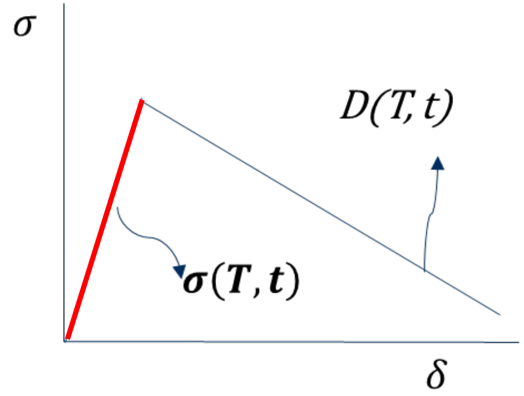
The loading part of the traction-separation curve is modelled by the **Generalized Maxwell model**

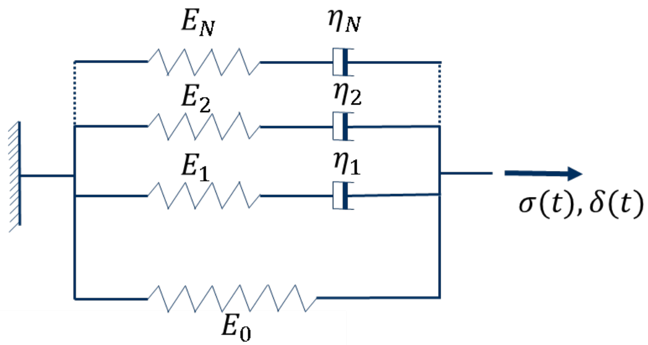
$$\sigma(T,t) = \int_0^t E_0 \dot{\delta}(s)ds + \sum_{i=1}^{N} \int_0^t E_i \exp\left(-\frac{t-s}{\tau_i}\right)\dot{\delta}(s)ds$$

Using FDM forward scheme, we get

$$\boxed{\sigma^{n+1} = \sigma_0^{n+1} + \sum_{i=1}^{N} \exp\left(-\frac{\Delta t}{\tau_i}\right)\sigma_i^n + \frac{E_i\tau_i}{E_0\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_i}\right)\right)\left[\sigma_{i0}^{n+1} - \sigma_{0i}^{n}\right]}$$

Traction-Separation curve





Rate- and Temperature-Dependent Viscoelastic CZM

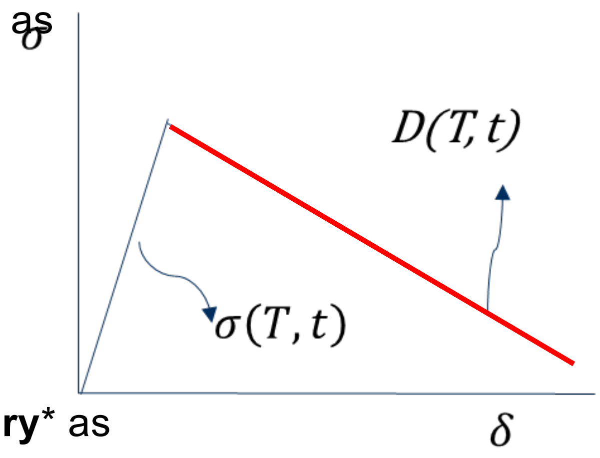
From sub-microcrack formation theory, damage of material could be represented as

$$D(T,t) = \frac{N(T,t)}{N_r}\ ; \ where \quad \begin{array}{l} D(T,t) = 0 \ \rightarrow no\ damage \\ D(T,t) = 1 \ \rightarrow macrocrack \end{array}$$

N_r- sub-microcrack concentration at rupture

Assuming a form for rate of damage

$$\frac{dD(T,t)}{dt} = \left(1 - D(T,t)\right)^p A(T,t)$$



where the damage rate constant A is represented by the **Zhurkov's kinetic theory*** as

$$\mathrm{A(T,t)} = \frac{1}{t_0}\exp((-U + \gamma\sigma(t))/(RT))$$

$$\boxed{\frac{dD(T,t)}{dt} = \left(1 - D(T,t)\right)^p \frac{1}{t_0}\exp((-U + \gamma\sigma(T,t))/(RT))}$$

Using FDM forward scheme

$$D^{n+1} = D^n + \Delta t(1 - D^n)^p \frac{1}{t_0}\exp((-U + \gamma\sigma^n)/(RT))$$

Finally, the traction is

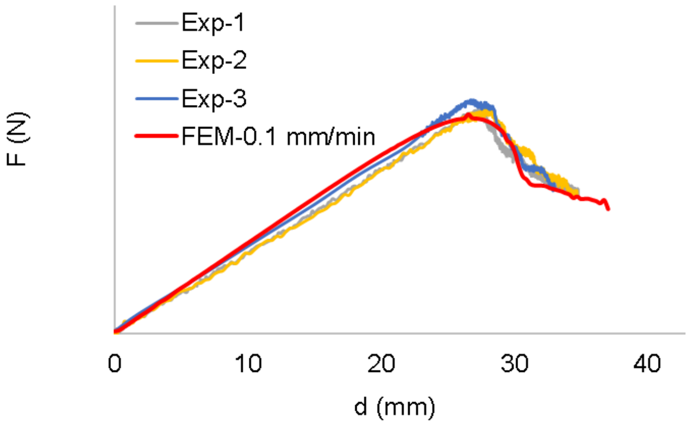
$$\sigma(T,t) = \left(1 - D(T,t)\right)^* \sigma(T,t)$$

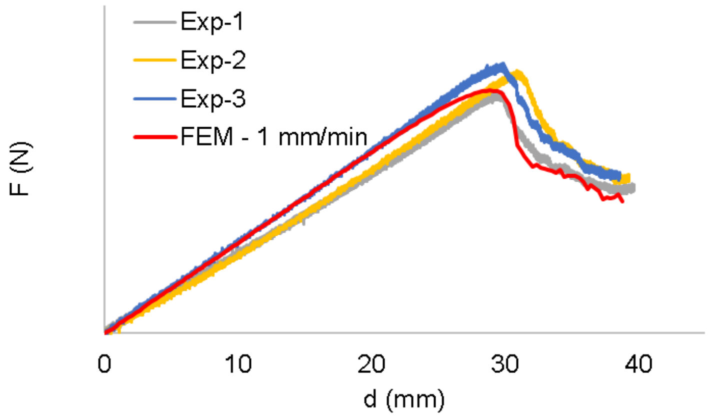
Implemented as material user subroutine ABAQUS /VUMAT

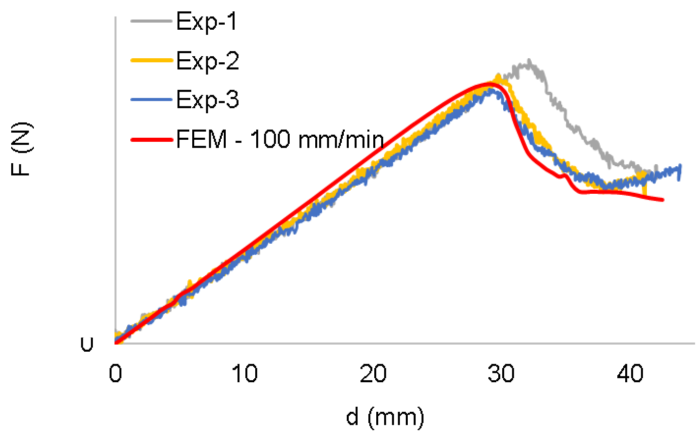
*A rate dependent kinetic theory of fracture for polymers, Hansan A. C and Baker-Jarvis J, International journal of fracture,1990.

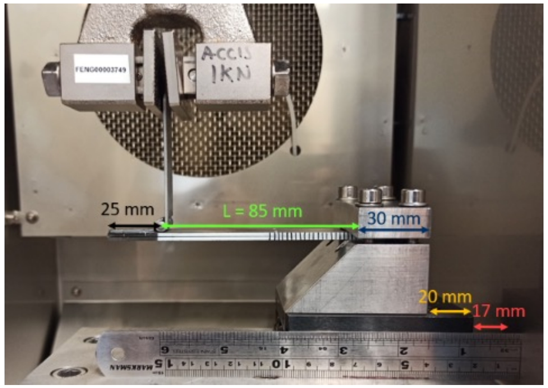
Rate- and Temperature-Dependent Viscoelastic – Mode II ELS

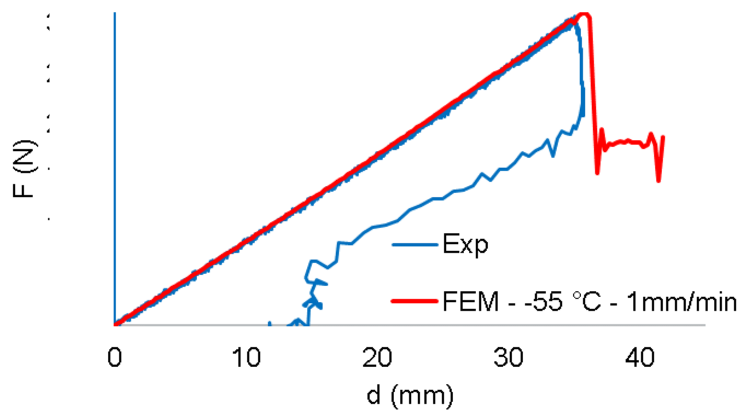
Model calibrated at room temperature, ASTM-standard rate, i.e. 1 mm/min

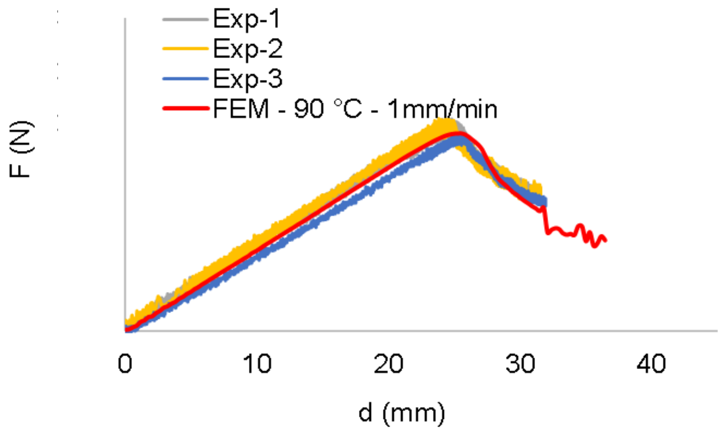








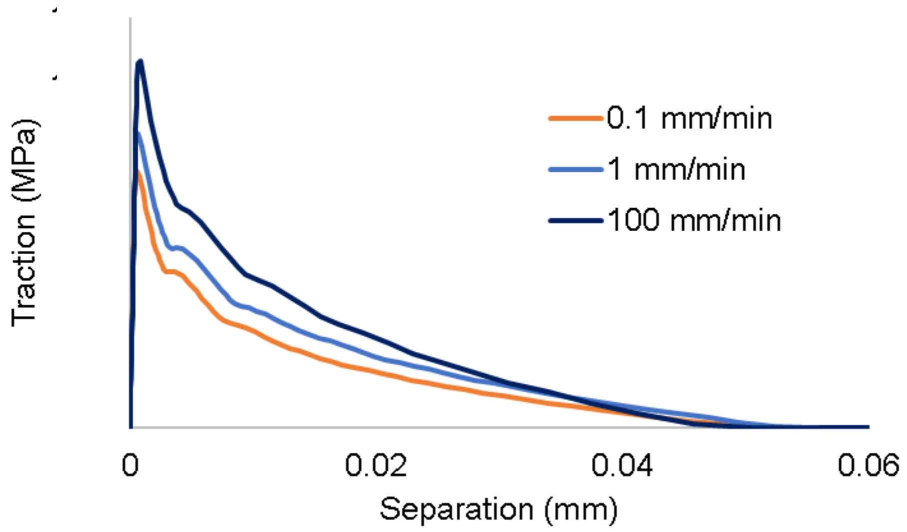




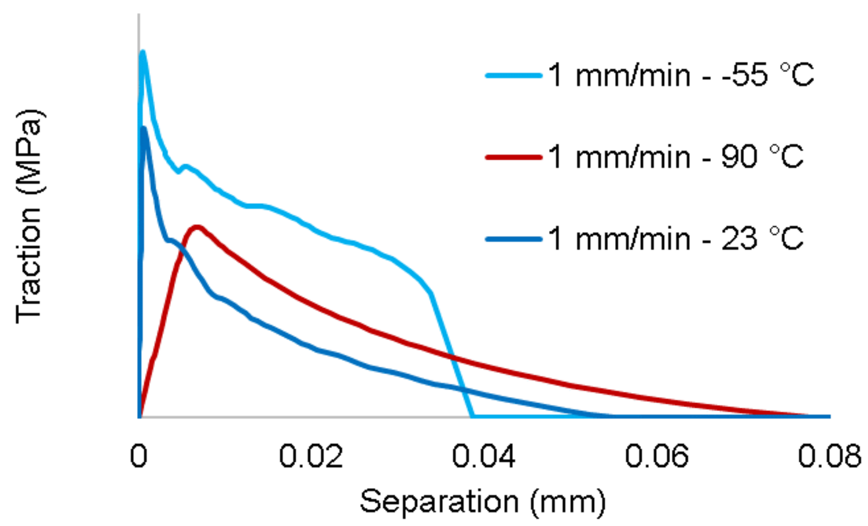
Rate- and Temperature-Dependent Viscoelastic – Mode II ELS

Traction-displacement laws from analysis – these are not prescribed, but obtained from the damage evolution law

Constant temperature 23 °C



Constant loading rate 5 mm/min



Thank you for your attention

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